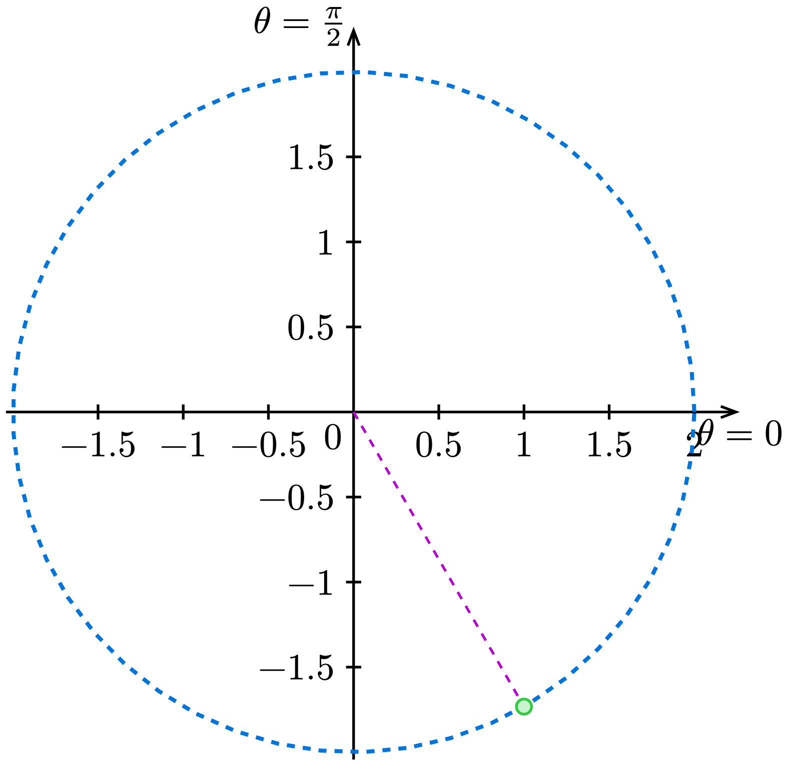
Polar coordinate

A polar coordinate $P(r, \theta)$ is represented by r, θ where r is the modulus and θ is the angle between OP and positive x-axis

e.g Plot $\left(2, \frac{5}{3}\pi\right)$ on polar coordinate.



The blue circle is $r = 2$ which is the set of all points that satisfies the conidition “distance from origin point is 2”.

The violet line has an angle of $\theta = \frac{5}{3}\pi / -\frac{\pi}{3}$ between positive x-axis and the line.

Hereby, $\theta \in (-\pi, \pi]$,$r > 0$

Note: Most graphing software (including desmos) accepts $r < 0$, this is not allowed in A-level exams except special circumstances where you plot the case $r < 0$ in dashed line.

Their intersection is the point on polar.

Relationship with Cartesian coordinate.

it can be easily observe that you can plot the same point in cartesian coordinate using

$$x^2 + y^2 = 4$$

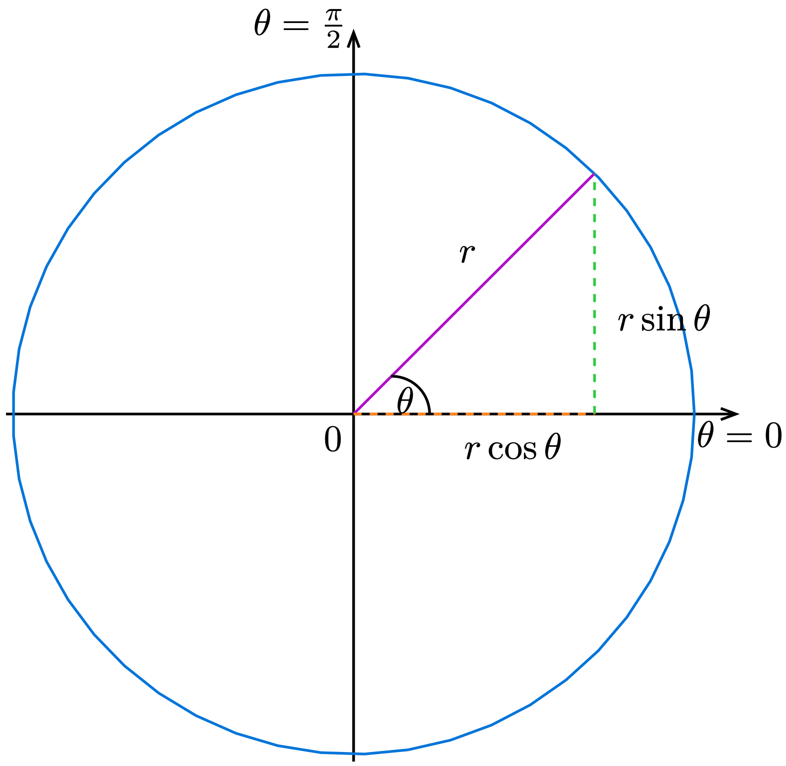
$$y = \tan\left(\frac{5\pi}{3}\right)x$$

More generally

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

By obeserving the unit circle we can have the transformation vice-versa.



Thus you can convert the polar coordinate into cartesian coordinate by using.

$$x = r \cos \theta$$

$$y = r \sin \theta$$

Curves

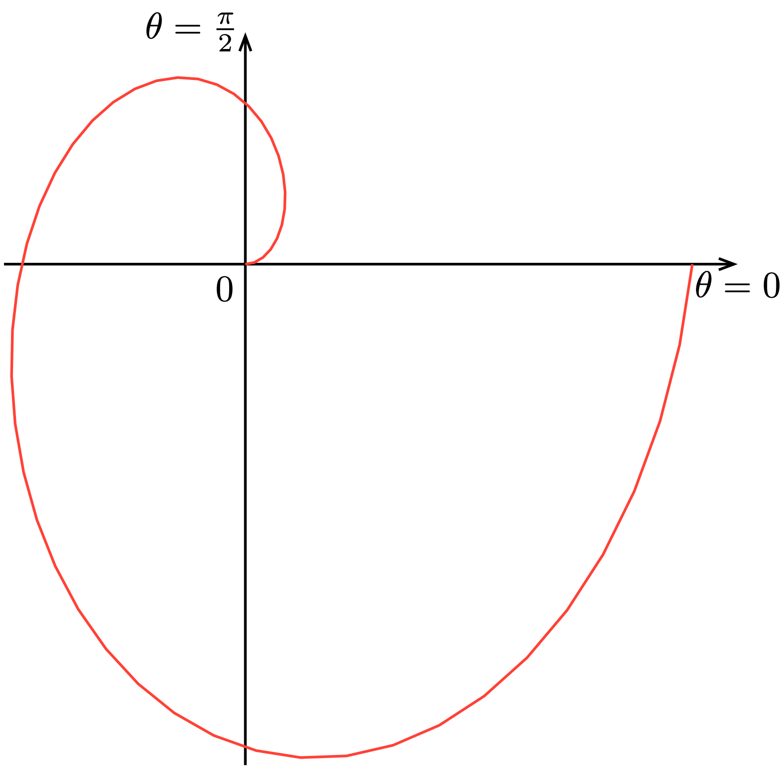
Recommended method to draw curves in the exam:

1. transform the polar curve into parametric curve
2. use calculator's table function to calculate the set of points.
3. connect the points using smooth curve.

Here are some common cases.

1. Sprials and its variations

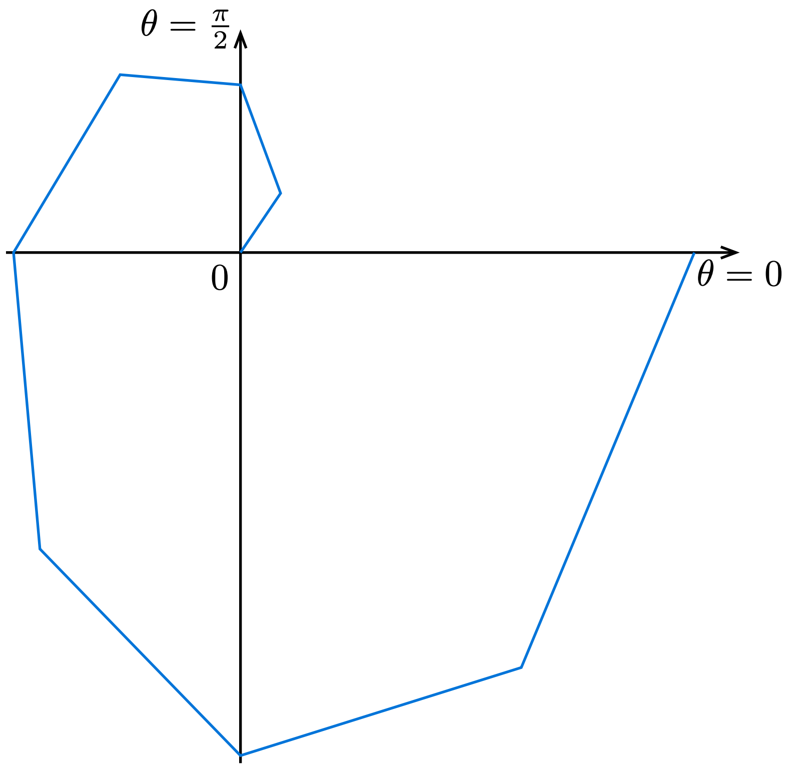
1. represented by $r = a\theta$ where a is a constant.



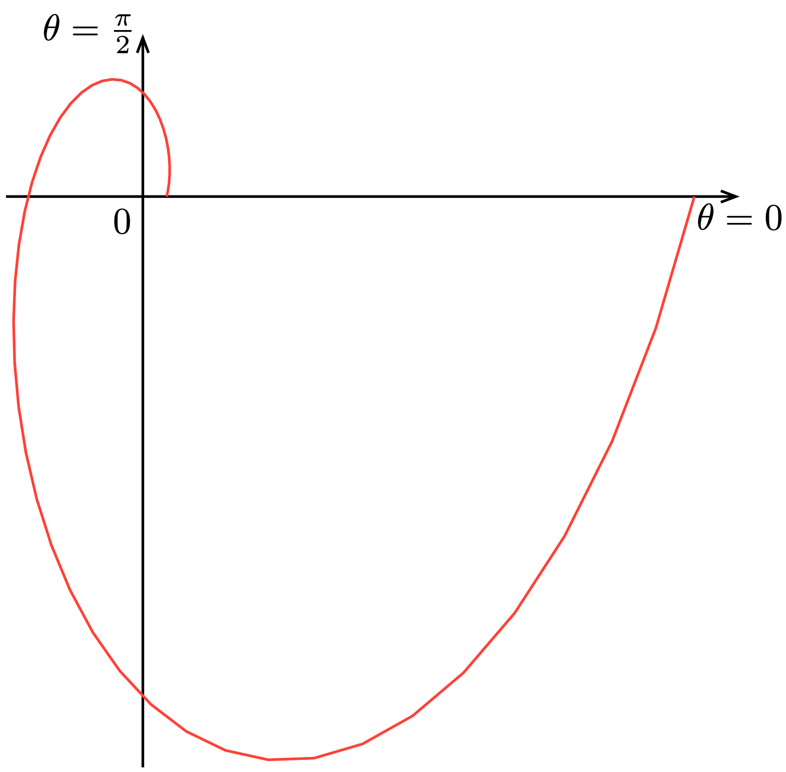
Example table (take $r = \theta$ as an example)

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
r	0	$\frac{\pi}{4}$	$\ldots$						

This is how it looks if you plot them on the coordinate and connect them with straight line.

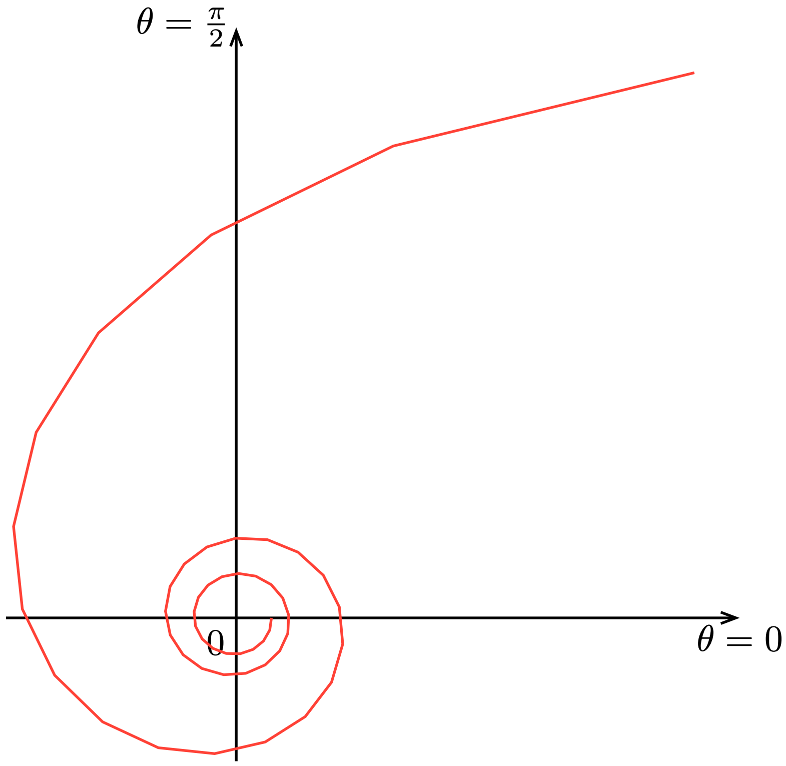


2. $r = ae^{k\theta}$



3. $r = \frac{a}{\theta}$

Here. $\lim_{\theta \to 0} r = \infty$

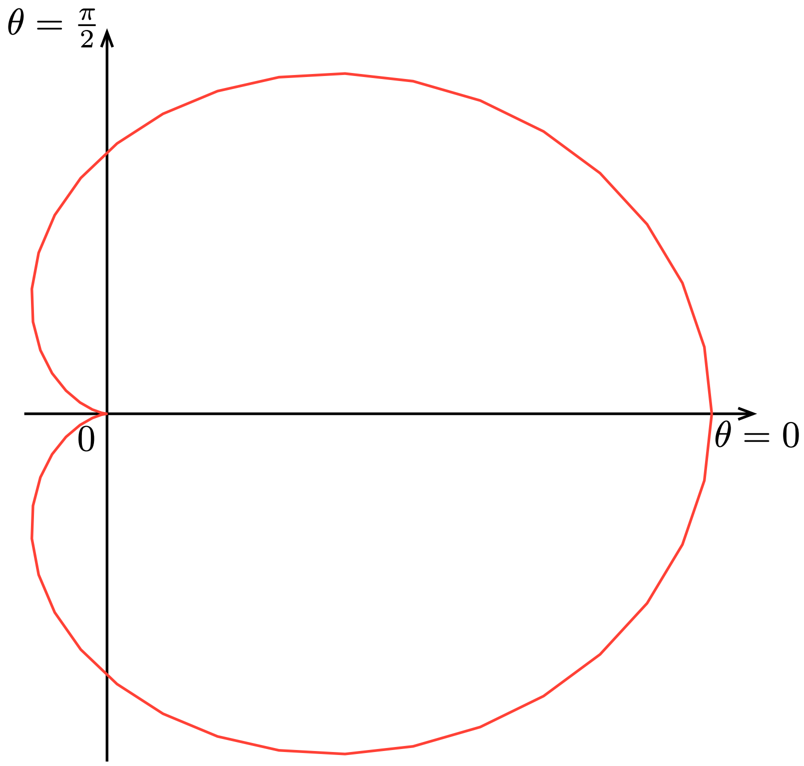


2. Cardioids and its variations

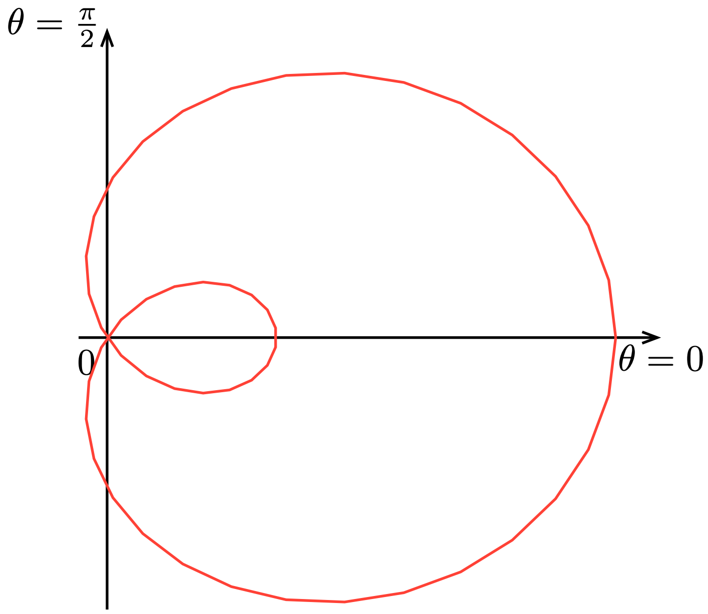
defined by $r = a + b\cos\theta$

Three cases, different cases the functions look different.

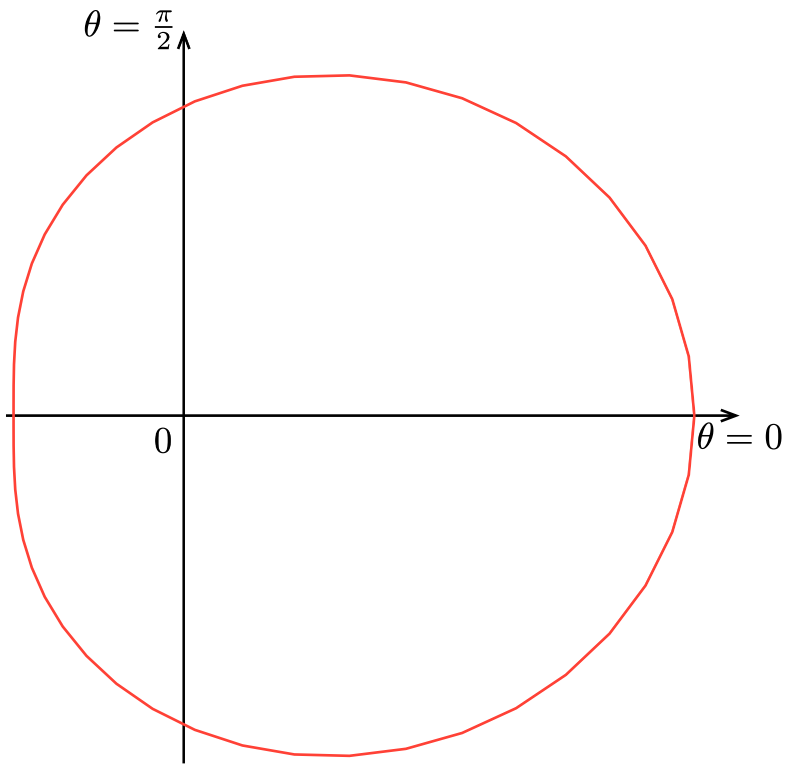
Case 1: $a = b$



Case 2: $a < b$

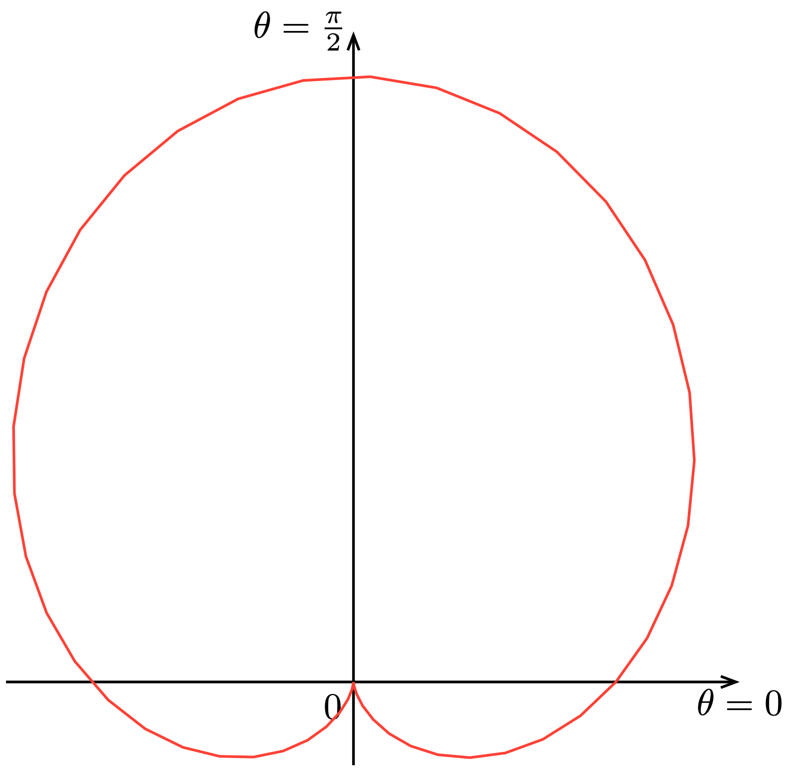


Case 3: $a > b$

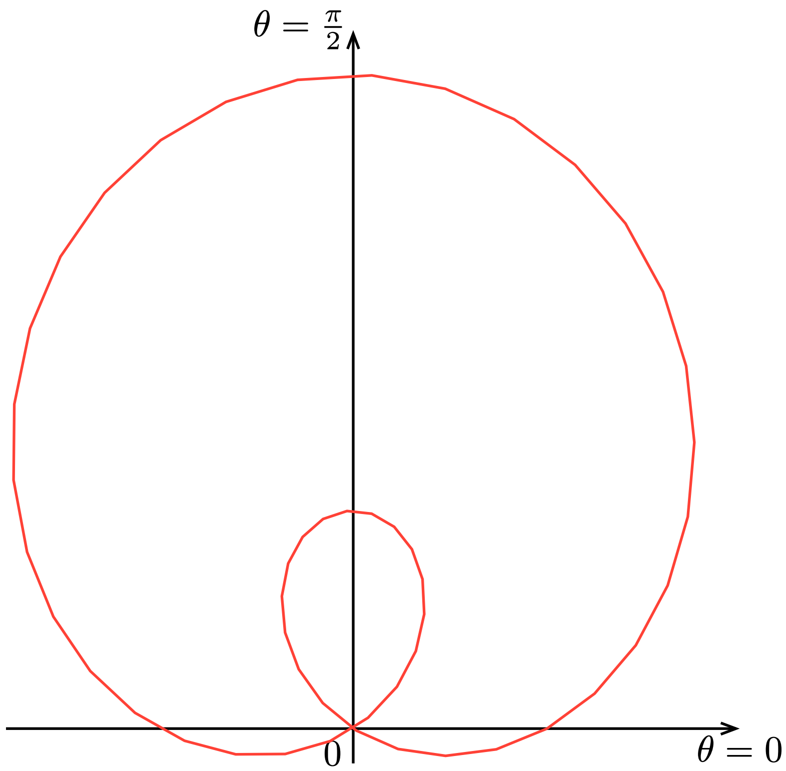


Its variation: $r = a + b\sin\theta$

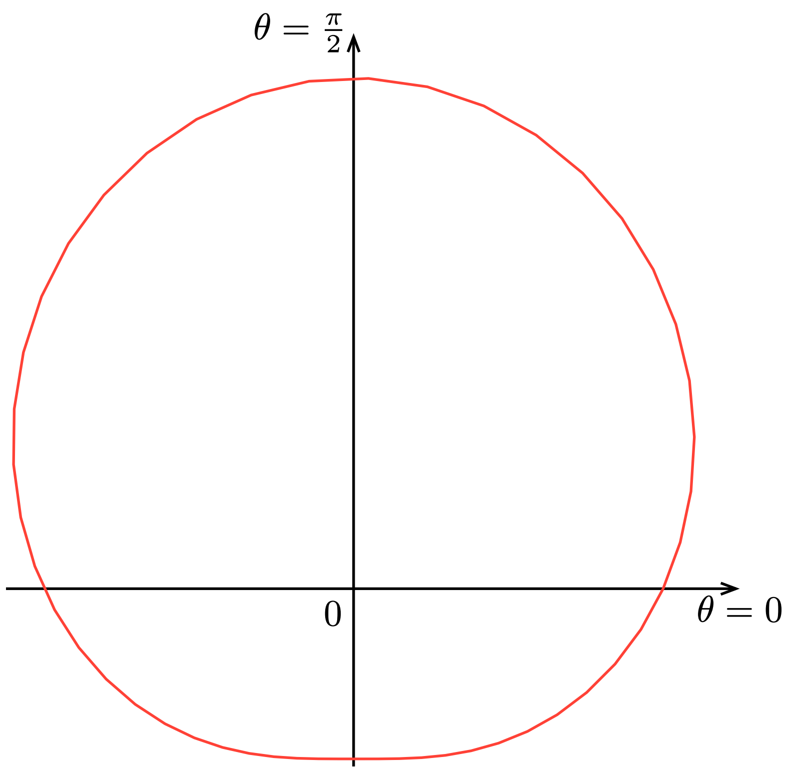
1. $a = b$



2. $a < b$



2. $a > b$



It's just turning from symmetrical to x-axis into symmetrical to y-axis.

Calculate surface area.

Recall the area of sector: $A = \frac{1}{2}r^2\theta$

Then the area is:

$$A = \int_{\theta_1}^{\theta_2} r^2 \, \mathrm{d}\theta$$

where $\theta_1 < \theta_2$

The area between to curves are:

$$A = \frac{1}{2} \int_{\theta_1}^{\theta_2} \left(r_2^2 - r_1^2\right) \, \mathrm{d}\theta$$

Finding Maximum/Mininum

it's actually just stationary point but polar version.

1. For finding maximum/minimum distance from the origin: use $\frac{\mathrm{d}r}{\mathrm{d}\theta} = 0$
2. For finding maximum/minimum distance from the $\theta = 0$ line: use $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 0$
3. For finding maximum/minimum distance from the $\theta = \frac{\pi}{2}$ line: use $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 0$